Discussion of Objectives

Unit 3. Polynomial and Rational Equations

Objective 3a. Use methods such as finding the greatest common factor, grouping, perfect squares, difference of squares, and sum and difference of cubes to factor trinomials and other polynomials completely.

Text sections and exercises

6.1, Greatest Common Factor and Factoring by Grouping, # 31 – 84
6.2, Factoring Trinomials: $x^2 + bx + c$, # 27 – 66
6.3, Factoring Trinomials: $ax^2 + bx + c$, # 1 – 66 (The $ac$-Method of Factoring)
6.4, Special Factoring Techniques, # 1 – 65

Software lessons

6.1b, Greatest Common Factor of a Polynomial
6.1c, Factoring Expressions by Grouping
6.3a, Factoring Trinomials by Trial and Error
6.3b, Factoring Trinomials by Grouping (The $ac$-Method of Factoring)
6.4a, Special Factorizations—Squares
6.4b, Special Factorizations—Cubes

Recommended software lessons for review

5.5, Multiplying Polynomials (Note that this software lesson is assigned.)
5.6a, The FOIL Method

The objective here is to use various methods to factor a polynomial completely—that is, to write it as a product of polynomials in which none of the factors can be factored further. Trinomials (polynomials with three terms) receive particular emphasis.

Factoring is not an exact science; one can only try different methods, and some polynomials just do not factor. Fortunately, it is easy to check whether a proposed factorization is correct by multiplying the factors to see if it yields the original polynomial. (To review multiplication of polynomials, see software lessons 5.5 and 5.6a.) The first step in factoring is to extract the greatest common factor (GCF) of the terms in the polynomial. A recommended next step is to see if the remaining expression is of a special form. Is it of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$? Is it a difference of two squares? A sum or difference of two cubes? In all those cases, familiarity with special factorizations allows one to factor it immediately. Often none of the forms applies, but the expression is a trinomial. In that case, one tries grouping (also called the $ac$-method) or trial and error. These methods can be slow but are more systematic than they might sound, and if they yield no factorization then the trinomial does not factor with integer coefficients. Finally, expressions with more than three terms are best treated by grouping.

Typical questions. Given a polynomial, you should be able to choose its complete factorization from among several given expressions or else select “Not factorable with integer coefficients.” Working with the text or software will expose you to problems that cover the entire range of factoring methods, including problems in which the polynomial is not factorable. Given a polynomial, and given one of the factors in a complete factorization of it, you should also be able to find the other factor, as in the following:
Example. Consider the polynomial $27x^3 + 8y^3$. One of the factors of this polynomial, when factored completely, is $3x + 2y$. Find the other factor.
Solution: $9x^2 - 6xy + 4y^2$

**Objective 3b. Solve polynomial equations by factoring.**

Text section and exercises
6.6, Solving Quadratic Equations by Factoring, # 1 – 74

Software lesson
6.6, Solving Quadratic Equations by Factoring

To solve a polynomial equation, one expands both sides if necessary, moves all terms to one side (so that the other side is zero), and factors the resulting expression. Starting with $x(x + 4) = 2x + 3$, for example, this yields the equation $(x - 1)(x + 3) = 0$. Since a product is zero exactly when one of the factors is zero, one obtains all solutions by equating the factors to zero, one at a time, and solving. In the example, the solutions are 1 and $-3$. Note that polynomial equations can have several solutions.

Typical questions. Given a polynomial equation, you should be able to carry out this process and obtain all solutions.

Example. Solve $x(x + 2)(3x - 5) = 0$.
Solution: 0, $-2$, and $5/3$; or, in set notation, $\{0, -2, 5/3\}$

Example. Consider the equation $x^2 + x = 25 + x$. Find two solutions.
Solution: $5$ and $-5$

**Objective 3c. Determine allowed values for rational expressions.**

Text section and exercises
7.1, Multiplication and Division with Rational Expressions, # 1 – 20 (state any restrictions only)

Software lesson
7.1a, Defining Rational Expressions

A rational expression is a fraction whose numerator and denominator are polynomials. Since division by zero is undefined, the variable in such an expression cannot equal any values that would cause the denominator to be zero. To find the “restricted values” of the variable—those for which the expression is not defined—one sets the denominator equal to zero and solves the resulting equation.

Typical questions. Given a rational expression, find the restricted values of the variable.

Example. Find the restricted values of $x$ for the rational expression $(2x + 8)/(x^2 + 3x - 10)$.
Solution: $x \neq -5$, $x \neq 2$

**Objective 3d. Solve equations involving rational expressions.**

Text section and exercises
7.4, Solving Equations with Rational Expressions, # 1 – 10, 29 – 56

Software lessons
7.4b, Solving Equations with Rational Expressions

Recommended software lessons for review

7.1b, Multiplication and Division of Rational Expressions
7.2, Addition and Subtraction of Rational Expressions

To solve an equation involving rational expressions, the main step is to multiply both sides by the least common denominator of the expressions that appear in it. This produces a polynomial equation, which one can solve using the methods of Objective 3b. A last step is to check the proposed solutions by inserting them into the original equation and discarding those which make a denominator zero.

Typical questions. Given an equation involving rational expressions, you should be able to give the least common denominator (LCD) of the terms in it, give the polynomial equation that results from multiplying both sides by that LCD, and give all solutions of the equation.

Example. Consider the equation $\frac{1}{x+6} - \frac{2}{x} = 1$. Give the least common denominator (LCD) of the expressions here. Then give the equation that results from multiplying both sides by that LCD. Finally, give all solutions of the original equation.

Solution: $(x+6)x; x - 2(x+6) = (x+6)x; -4, -3$

Example. Solve the equation $\frac{1}{x-1} = 3/x + x/(x-1)$.

Solution: $-3$, (not 1, since it makes a denominator 0)

**Objective 3e. Use rational equations to solve applied problems, including work and rate problems.**

Text section and exercises

7.5, Applications, # 1 – 35

Software lesson

7.5, Applications Involving Rational Expressions

Like other objectives directed at applications, this one addresses all the steps in solving an applied problem: setting up an equation that captures the given information, solving it, and interpreting the mathematical solution as it applies to the original problem. Again, the hardest step is to set up the equation. The key is to assign a variable name to the quantity that one is trying to find, and then focus on expressing the given information in terms of the variable. Rational expressions arise in several situations. For instance, if a question asks for the speed $x$ at which a cyclist rides and the given information concerns the time that it takes to ride 20 miles, then one can express that time as $20/x$. If a question asks for the number of hours $t$ that it takes Madeline to paint a room and the given information concerns the rate at which she works, then one can express Madeline’s contribution to the total rate as $1/t$ rooms per hour.

Typical questions. Each exam question addresses either just one part of the process (setting up an equation in an applied problem or using an equation that is already set up to solve the problem) or it addresses the entire process.

Example. Rainsby takes three times as long as Claude to mow a lawn. Working together, they can mow the lawn in 75 minutes. Give an equation that one could solve to determine the number of minutes $t$ that it takes Claude to mow the lawn.
Solution: $1/t + 1/(3t) = 1/75$

Example. Daphne rows 4 mph in still water. On a certain river, it takes her the same time to row 3 miles upstream as it takes to row 7 miles downstream. Let $x$ be the speed of the current. The given information can be summarized in the equation $3/(4 - x) = 7/(4 + x)$. Find the speed of the current. Round your answer to the nearest tenth of a mile per hour.
Solution: 1.6 mph

Example. Spode takes twice as long as Lady Wickham to paint a room. Together they can paint the room in 2 hours. How long would it take Lady Wickham to paint it by herself?
Solution: 3 hours